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TECHNICAL NOTES

Film condensation along a frustum of a cone in a porous medium

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NOMENCLATURE

c_p	specific heat of the fluid
f	dimensionless stream function
g	acceleration due to gravity
h_x	local heat-transfer coefficient
h_{fg}	latent heat of vaporization
K	permeability of the porous medium
k_m	thermal conductivity of the saturated porous medium
$\dot{m}$	mass flux across the interface
Nu_x	local Nusselt number
q	local heat-transfer rate
r_0	radius of the cone
Ra	Rayleigh number
Ra_x	local Rayleigh number
Sc	dimensionless degree of wall subcooling
T	temperature
u, v	Darcy's velocities in x- and y-directions, respectively
$\bar{u}, \bar{v}$	dimensionless Darcy's velocities
U_L	reference velocity
x, y	distances along and perpendicular to the surface
$\bar{x}, \bar{y}$	dimensionless distances.

Greek symbols

α	equivalent thermal diffusivity
δ	boundary-layer thickness
θ	dimensionless temperature
μ	viscosity of the fluid
ξ, η	transformed coordinates
ρ	density of the fluid
ϕ	cone angle
ψ	stream function.

Superscript

$'$	denotes differentiation with respect to η_L.

Subscripts

L	liquid phase
s	saturated condition
v	vapour phase
w	conditions at the wall
$x, \bar{x}, \bar{y}, \xi$	denote derivatives with respect to x, $\bar{x}$, $\bar{y}$ and ξ, respectively.

1. INTRODUCTION

THE TWO-phase flow problems in a porous medium with phase change have important applications in geothermal energy utilization [1], thermal enhancement of oil recovery [2], etc. The classical film condensation problem has been studied by Sparrow and Greeg [3], Koh *et al.* [4], and Merte [5]. Recently, Cheng [6] has investigated the problem of steady film condensation outside a wedge or a cone embedded in a porous medium filled with dry saturated vapour.

In this note, the problem of steady film condensation along a frustum of a cone without transverse curvature effect (i.e. large cone angles when the boundary-layer thickness is small compared with the local radius of the cone) has been considered. The governing equations have been solved numerically using Keller's box method [7, 8]. The results have been compared with those of Cheng [6].

2. GOVERNING EQUATIONS

Consider a frustum of a cone (having a semi-vertical angle ϕ) with constant wall temperature T_w which is embedded in a porous medium filled with a dry saturated vapour at a saturated temperature T_s (corresponding to its pressure). It is assumed that: (a) the condensate and the vapour are separated by a distinct boundary; (b) the condensate has constant properties; and (c) condensate film is thin such that boundary-layer approximations are applicable. If the wall temperature T_w is less than the saturated temperature T_s, a film of condensate will form adjacent to the surface and flow downward due to gravity. The physical model and coordinate system is shown in Fig. 1. The governing partial differential equations with boundary-layer approximations can be

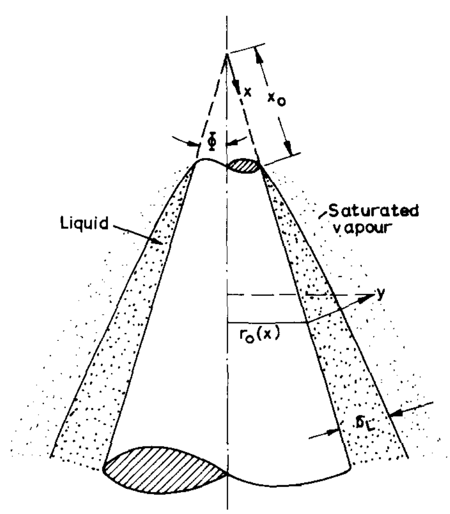

FIG. 1. Coordinate systems for film condensation.

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expressed in dimensionless form as [6, 9]

$$(\bar{r}_0\bar{u}_L)_{\bar{x}}+(\bar{r}_0\bar{v}_L)_{\bar{y}} = 0 \tag{1}$$

$$\bar{u}_L = 1 \tag{2}$$

$$\bar{u}_L(\theta_L)_{\bar{x}}+\bar{v}_L(\theta_L)_{\bar{y}} = (\theta_L)_{\bar{y}\bar{y}} \tag{3}$$

subject to the boundary conditions at the wall

$$\bar{y} = 0: \quad \bar{v}_L = 0, \quad \theta_L = 1 \tag{4}$$

and interface conditions

$$\bar{y} = \bar{\delta}_L: \quad \theta_L = 0, \quad [\bar{u}_L(\bar{\delta}_L)_{\bar{x}}-\bar{v}_L]_{\bar{y}=\bar{\delta}_L} = -Sc[(\theta_L)_{\bar{y}}]_{\bar{y}=\bar{\delta}_L} \tag{5}$$

where

$$\begin{aligned} &\bar{x} = (x-x_0)/x_0, \quad \bar{y} = (y/x_0)Ra^{1/2}, \\ &\bar{u}_L = u_L/U_L, \qquad \bar{v}_L = (v_L/U_L)Ra^{1/2} \\ &\theta = (T_L-T_s)/(T_w-T_s), \quad \bar{r}_0 = r_0(x)/x_0, \\ &\bar{\delta}_L = (\delta_L/x_0)Ra^{1/2} \end{aligned} \tag{6a}$$

$$\begin{aligned} &U_L = K(\rho_v-\rho_L)g\cos\phi/\mu_L, \quad Ra = U_L x_0/\alpha_L \\ &Sc = c_{pL}(T_w-T_s)/h_{fg}, \qquad r_0 = x\sin\phi. \end{aligned} \tag{6b}$$

We apply the following transformations

$$\begin{aligned} &\xi = \bar{x}, \quad \eta_L = \bar{y}/\xi^{1/2}, \quad f_L(\xi,\eta_L) = \xi^{-1/2}\bar{r}_0^{-1}\psi_L \\ &g_L(\xi,\eta_L) = \theta_L, \quad \bar{r}_0\bar{u}_L = (\psi_L)_{\bar{y}}, \quad \bar{r}_0\bar{v}_L = -(\psi_L)_{\bar{x}} \end{aligned} \tag{7}$$

to equations (1)–(3) and we find that equation (1) is identically satisfied and equations (2) and (3) reduce to

$$f'_L = 1 \tag{8}$$

$$g''_L+[\xi(1+\xi)^{-1}+2^{-1}]f_Lg'_L = \xi[f'_L(g_L)_\xi-g'_L(f_L)_\xi]. \tag{9}$$

Conditions (4) and (5) reduce to

$$f_L(\xi,0) = 0, \quad g_L(\xi,0) = 1 \tag{10a, b}$$

$$g_L(\xi,\eta_{L\delta}) = 0 \tag{11a}$$

$$Sc\, g'_L(\xi,\eta_{L\delta}) = -[\xi(1+\xi)^{-1}+2^{-1}]f_L(\xi,\eta_{L\delta}). \tag{11b}$$

The integration of equation (8) with the condition given in equation (10a) gives

$$f_L = \eta_L. \tag{12}$$

Using equation (12), equation (9) can be expressed as

$$g''_L+[\xi(1+\xi)^{-1}+2^{-1}]\eta_Lg'_L = \xi(g_L)_\xi. \tag{13}$$

Similarly, equation (11b) reduces to

$$Sc\, g'_L(\xi,\eta_{L\delta}) = -[\xi(1+\xi)^{-1}+2^{-1}]\eta_{L\delta}. \tag{14}$$

It may be remarked that for $\xi = 0$ and $\xi \to \infty$, the partial differential equation, equation (13), reduces to the similarity equation for flow over a wedge and a full cone, respectively, which have been studied by Cheng [6]. These equations are

$$\begin{aligned} &g''_L+(1/2)\eta_Lg'_L = 0 \quad \text{when } \xi = 0 \\ &g''_L+(3/2)\eta_Lg'_L = 0 \quad \text{when } \xi \to \infty. \end{aligned} \tag{15}$$

We now consider the flow field in the vapour phase at $y > \delta_L$. Since the vapour is at a constant temperature T_s, the energy equation is automatically satisfied. With the help of the continuity equation and the boundary-layer approximations applied to Darcy's law, we have [6]

$$u_v = 0, \quad v_v = f(x). \tag{16}$$

From the interface mass continuity equation [6]

$$\dot{m} = \rho_v[u_v(\delta_v)_x-v_v]_{y=\delta_L} = \rho_L[u_L(\delta_L)_x-v_L]_{y=\delta_L} \tag{17}$$

and using equations (6) and (7), the dimensionless vapour velocity can be expressed as

$$(Ra)^{-1/2}\bar{v}_v = -[\xi(1+\xi)^{-1}+2^{-1}]\xi^{-1/2}\eta_{L\bar{\delta}} \tag{18}$$

where

$$\bar{v}_v = (\rho_v v_v)/(\rho_L U_L). \tag{19}$$

This relation indicates that the vapour is moving towards the interface.

The heat-transfer rate is given by

$$\begin{aligned} q_w &= -k_{mL}(\partial T/\partial y)_{y=0} \\ &= -k_{mL}(T_w-T_s)Ra^{1/2}x_0^{-1}\xi^{-1/2}g'_L(\xi,0) \\ &= h_x(T_w-T_s). \end{aligned} \tag{20}$$

Now the heat transfer in terms of the Nusselt number is given by

$$Nu_{x*} = (h_x x^*/k_{mL}) = Ra_{x*}^{1/2}[-g'_L(\xi,0)] \tag{21}$$

where

$$x^* = x-x_0, \quad Ra_{x*} = U_L x^*/\alpha_L. \tag{22}$$

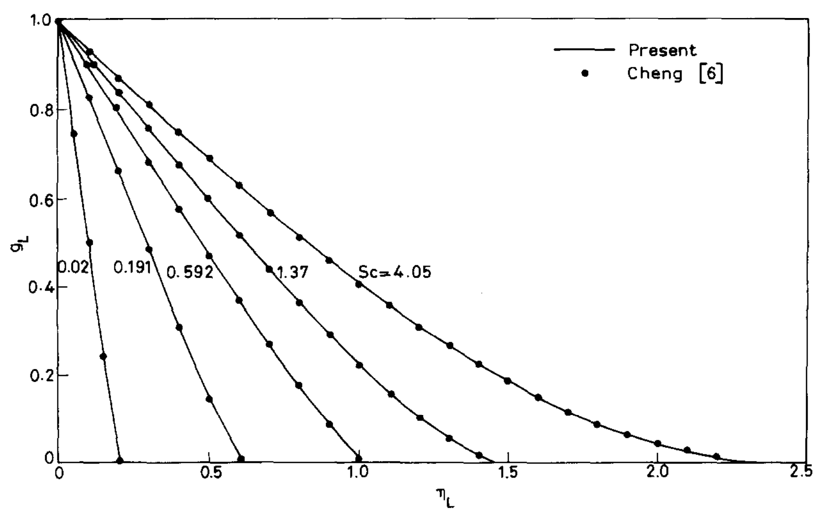

FIG. 2. Comparison of temperature results.

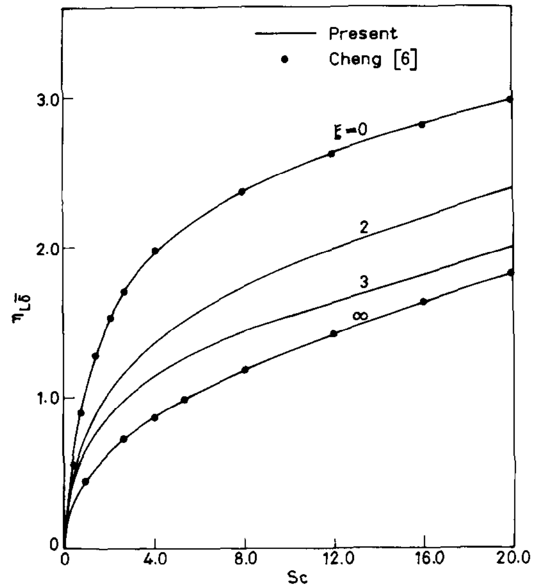

FIG. 3. Variation of film thickness with wall subcooling parameter and streamwise distance.

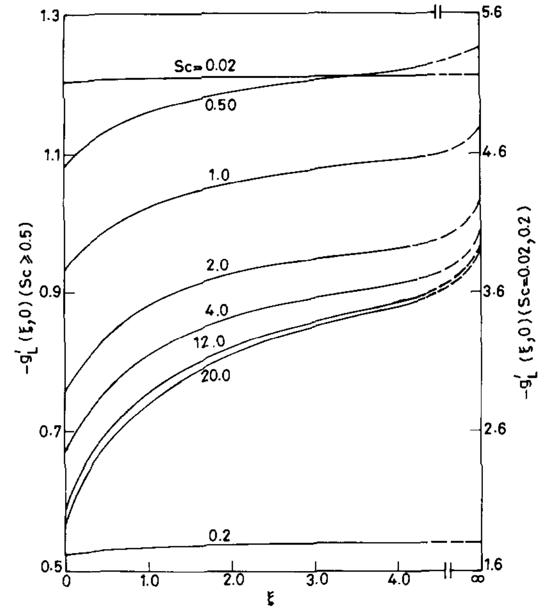

FIG. 4. Variation of heat-transfer parameter with streamwise distance and wall subcooling parameter.

3. RESULTS AND DISCUSSION

Equation (13) under boundary conditions (10b), (11a) and (14) has been solved numerically using a finite-difference scheme developed by Keller [7]. Since the method is described in great detail in refs. [7, 8], it is not presented here. We have studied the effect of step sizes $\Delta\eta$ and $\Delta\xi$ and the edge of the boundary layer (η_∞) on the solution with a view to optimize them. Consequently, computations were carried out on a DEC-1090 computer with $0.0005 \leqslant \Delta\eta \leqslant 0.0009$, $\Delta\xi = 0.1$, $0.2 \leqslant \eta_\infty \leqslant 3.6$ depending on the values of Sc. The results presented here are independent of the step sizes and η_∞ at least up to fourth decimal place. The CPU time taken by a typical data is 5.4 s.

In order to assess the accuracy of the method for the problem under consideration, we have compared our results for dimensionless temperature (g_L) in the condensate and the thickness of the liquid film ($\eta_{L\delta}$) for $\xi = 0$ (wedge case) and $\xi \to \infty$ (full cone case) with those of Cheng [6]. The comparison is given in Figs. 2 and 3. However, to reduce the number of curves in Fig. 2, the comparison is shown only for $\xi = 0$. The results are found to be in very good agreement.

The variation of the heat-transfer parameter for film condensation ($-g'_L(\xi, 0)$) with ξ for various values of the wall subcooling parameter (Sc) is shown in Fig. 4. Also the variation of the thickness of the liquid film $\eta_{L\delta}$ with the wall subcooling parameter (Sc) for several values of ξ is shown in Fig. 3. It is observed that for a given ξ, the heat-transfer parameter ($-g'_L(\xi, 0)$) decreases as Sc increases. The effect is more pronounced for small values of $Sc (Sc \leqslant 0.2)$. This is due to the increase in the thickness of the liquid film ($\eta_{L\delta}$) as Sc increases (Fig. 3) which causes reduction in the heat transfer. Also for a given Sc, $-g'_L(\xi, 0)$ increases with ξ (Fig. 3) due to the reduction in the thickness of the liquid film as ξ increases (Fig. 3). However, for small values of $Sc (Sc \leqslant 0.2)$, $-g'_L(\xi, 0)$ changes very little with ξ. Since $\xi = 0$ corresponds to the similarity solution of the wedge case and $\xi \to \infty$ to the similarity solution of a full cone case, the heat transfer ($-g'_L(\xi, 0)$) is higher for the full cone than for the wedge. This is true for all values of Sc except when it is very small ($Sc \leqslant 0.2$). In that case, the change is very small. On the other hand, the thickness of the liquid film is less for the full cone than for the wedge (see Fig. 3). Similar effects have been observed by Cheng [6].

The dimensionless temperature profile in the condensate (g_L) for various values of Sc and ξ has also been obtained, but is not presented here for the sake of brevity. The temperature profile g_L becomes less steep as Sc increases. The effect of ξ is just the opposite. This is due to the fact that the thickness of the liquid film increases as Sc increases, but it decreases with ξ.

4. CONCLUSIONS

The heat transfer is found to be strongly dependent on the subcooling parameter when its value is comparatively small. For large values of the subcooling parameter, the effect is rather small. On the other hand, it is weakly dependent on the streamwise distance when the subcooling parameter is very small. Also the thickness of the liquid film increases with the subcooling parameter, but decreases with the streamwise distance.

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